

# Considerations on the influence of the moment of inertia on the movement of a vehicle

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**Abstract.** This paper presents a mathematical model for automotive stability study taking into account the influences of the inertias/s moment. It is also presents an experimental method for determination of the inertias/s moment for a vehicle related to the axis of a system which has the origin in its mass centre. The used procedure is based on the pendulous method which consists in the vehicle/s postponement and the measurement of the oscillations period around the suspension axle. The inertia moments of the tested vehicle are determined experimentally for three cases of loading, taking into account the load of a passenger case, three passengers case and five passengers case. The numerical results are determined based on a simplified mechanical model and a nonlinear differential mathematical model. It is investigated several cases of vehicle movement with different load and thus is highlighted the influence of different moments of inertia over the dynamic behaviour of the considerate vehicle.

## 1. Introduction

The assessment of the dynamic and cinematic behavior of a car is the starting point in the conception phase of that. For this purpose, the mechanical models and mathematical models is designed according to sizes of interest. Mechanical models have different degrees of approximation of the real model [2], [5], [9] and lead to mathematical models with different levels of complexity [3], [4], [8].

## 2. Theoretical considerations on the method of determining moment of inertia

It is considered a rigid solid with a horizontal fixed axis, subjected only to the action of the gravitational field (Figure 1), relative to two reference systems, as follows:

- a fixed outer reference system (E) having the axes  $Ox_1y_1z_1$ , the axis  $Ox_1$  being the vertical of the place;
- an own reference system (P) connected to a rigid system, with  $Oxyz$  axes, the  $X$  axis passing through the center of mass of the rigid body.

The equation of motion of the solid, in case of friction neglect, is:

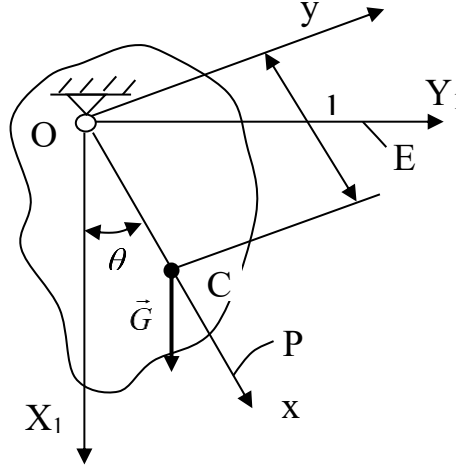
$$J_o \ddot{\theta} + Gl \sin \theta = 0 \quad (1)$$

where: -  $J_o$  is the moment of inertia of the solid to the oscillation axis of the suspension;

-  $\theta$  is the angle between  $Ox_1$  -  $Ox$  axes;

-  $G$  is the weight of the body;

-  $l$  is the distance from the suspension axis to the center of gravity of the solid;



**Figure 1.** The mechanical model of the solid to determine the moment of inertia

In the case of small oscillations equation (1) takes the form:

$$J_0 \ddot{\theta} + Gl\theta = 0 \quad (2)$$

The period of small oscillations is given by:

$$T = 2\pi \cdot \sqrt{\frac{J_0}{G \cdot l}} \quad (3)$$

Relationship (3) shows that small oscillations are autocrons, the oscillation period does not depend on amplitude.

In case of higher oscillations the period of movement is:

$$T = 2\pi \cdot \sqrt{\frac{J_0}{G \cdot l} \left( 1 + \frac{\alpha^2}{16} \right)} \quad (4)$$

where:  $\alpha$  is the angular oscillation of the oscillations.

Therefore, to determine the moment of inertia  $J_0$ , the weight of the solid  $G$ , the position of the center of gravity (through the distance  $l$  from the center of the suspension) and the period of oscillations,  $T$ .

The moment of inertia of the solid to the axis of oscillation takes the form:

- for small oscillations:

$$J_0 = \frac{T^2 \cdot G \cdot l}{4\pi^2} \quad (5)$$

- for large amplitude oscillations:

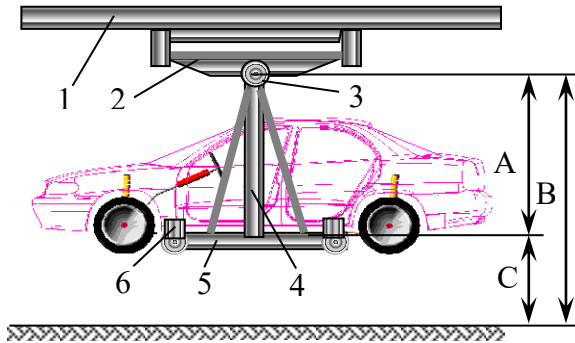
$$J_0 = \frac{T^2 \cdot G \cdot l}{4\pi^2 \left( 1 + \frac{\alpha^2}{16} \right)} \quad (6)$$

The moment of inertia of the solid to an axis passing through the center of mass and parallel to the suspension axis is calculated using Steiner's theorem:

$$J_c = J_0 - \frac{Gl^2}{g} \quad (7)$$

### 3. Experimental results

The installation for determining the moment of inertia consists of a frame on which the vehicle is fixed, a frame which can be suspended by a beam, in order to allow it to oscillate (Figure 2).



**Figure 2** Stand for determining moment of inertia

1 - supporting beam; 2 - horizontal shaft assembly (having axis parallel to the  $O_x$  axis) made with hinges (radial ball bearings); 3 - horizontal shaft assembly (with axis parallel to the  $O_y$  axis) rigidly fixed by the perpendicular shaft 2, also made with ball radial ball bearings; 4 - vertical support; 5 - car support frame; 6 - car fixation device.

Several test cycles were performed under the following conditions:

-the following angular amplitudes  $\alpha$  and loads are intended:

$\alpha=10^\circ$  – without load     $\alpha=10^\circ$  – 1 passenger load     $\alpha=10^\circ$  – 2 passengers load     $\alpha=10^\circ$  – 5 passengers

$\alpha=15^\circ$  – without load     $\alpha=15^\circ$  – 1 passenger load     $\alpha=15^\circ$  – 2 passengers load     $\alpha=15^\circ$  – 5 passengers

$\alpha=20^\circ$  – without load     $\alpha=20^\circ$  – 1 passenger load     $\alpha=20^\circ$  – 2 passengers load     $\alpha=20^\circ$  – 5 passengers

-at least 30 oscillations shall be made at each test.

The results of experimental research and their average are presented in the table below.

| Moment of inertia |                                     | Angle $\alpha$ |      |      | Average |
|-------------------|-------------------------------------|----------------|------|------|---------|
|                   |                                     | 10°            | 15°  | 20°  |         |
| Without load      | $J_{xx}^{(0)}$ [kg.m <sup>2</sup> ] | 262            | 525  | 525  | 394     |
|                   | $J_{yy}^{(0)}$ [kg.m <sup>2</sup> ] | 1307           | 2614 | 2614 | 1961    |
|                   | $J_{zz}^{(0)}$ [kg.m <sup>2</sup> ] | 1450           | 2901 | 2901 | 2176    |
| 1 passenger       | $J_{xx}^{(0)}$ [kg.m <sup>2</sup> ] | 278            | 556  | 556  | 417     |
|                   | $J_{yy}^{(0)}$ [kg.m <sup>2</sup> ] | 1384           | 2768 | 2768 | 2076    |
|                   | $J_{zz}^{(0)}$ [kg.m <sup>2</sup> ] | 1536           | 3072 | 3072 | 2304    |
| 2 passengers      | $J_{xx}^{(0)}$ [kg.m <sup>2</sup> ] | 293            | 586  | 586  | 440     |
|                   | $J_{yy}^{(0)}$ [kg.m <sup>2</sup> ] | 1461           | 2922 | 2922 | 2192    |
|                   | $J_{zz}^{(0)}$ [kg.m <sup>2</sup> ] | 1620           | 3242 | 3242 | 2432    |
| 5 passengers      | $J_{xx}^{(0)}$ [kg.m <sup>2</sup> ] | 393            | 786  | 786  | 590     |
|                   | $J_{yy}^{(0)}$ [kg.m <sup>2</sup> ] | 1689           | 3378 | 3378 | 2534    |
|                   | $J_{zz}^{(0)}$ [kg.m <sup>2</sup> ] | 1874           | 3748 | 3748 | 2811    |

The values of these moments are used for the dynamic study of vehicles, namely for the analysis of their stability, and it is easy to see that their value is dependent on the load size and the maximum values are those based on the  $O_z$  axis.

#### 4. Numerical results

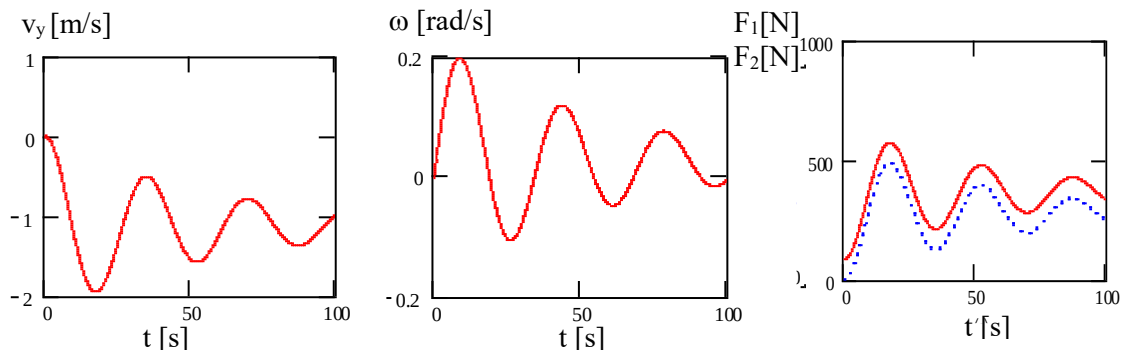
For the displacement after a circular trajectory, the obtained solutions will be presented graphically considering for each of them two different values of the moment of inertia, corresponding to the case when the vehicle moves only with the load of the driver or with a load of five persons.

The following numeric values will be used:

- tire rigidity  $k_1 = k_2 = 250 \text{ N / grad}$
- wheelbase  $A = 2570 \text{ mm}$ .

For the case of one passenger car the following parameters are used: distance from center of gravity to front axle  $a = 1113 \text{ mm}$ , distance from centre of gravity to rear axle  $b = 1457 \text{ mm}$ , mass  $m = 1358 \text{ kg}$ , moment of inertia  $I = 2611 \text{ kgm}^2$ ;

For the case of circular trajectory [6] and one passenger load the next numerical results are obtained:

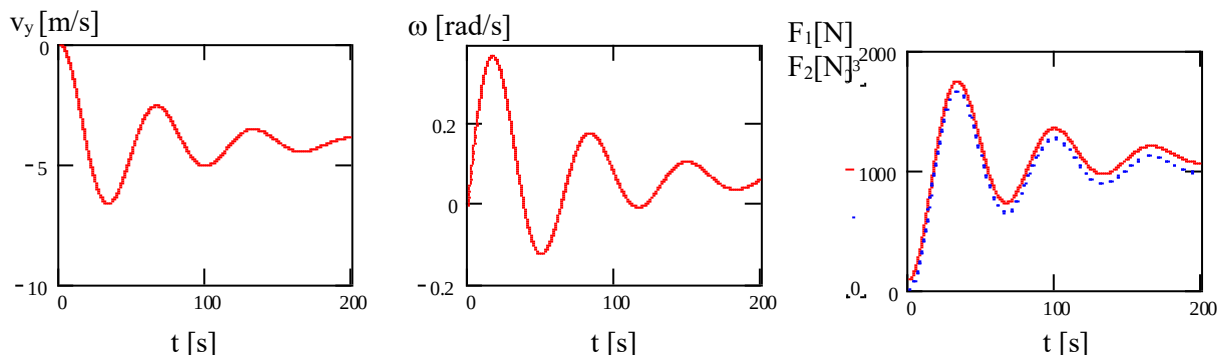


**Figure 3.** Numerical results in case of one passenger load

For the case of five passengers car the following parameters are used: distance from centre of gravity to front axle  $a = 1234 \text{ mm}$ , distance from centre of gravity to rear axle  $b = 1336 \text{ mm}$ , mass  $m = 1667 \text{ kg}$ , moment of inertia  $I = 2802 \text{ kgm}^2$ ;

In this case the cars movement is stable, the amplitude of the angular velocity is  $0.2 \text{ rad / sec}$  and the wheel force amplitude is approximately 600.

For the case of circular trajectory and five passengers load the next numerical results are obtained:



**Figure 4.** Numerical results in case of five passengers load

The movement has the same characteristics, it is stable, the magnitude of the rotation speed is  $0.4 \text{ radian/sec}$  and the wheel force amplitude is approximately 1700 N.

#### 5. CONCLUSION

This mathematical model is a simple but useful model for vehicle stability analysis, based on the theory of dynamic systems or the theory of classical mechanics. The values obtained are accurate values,

compared to the measurements made on a real vehicle [7], having the same parameters as those used in the numerical simulation, so this can be considered a validation for this model.

For the proposed model and for the types of motion analyzed, the variation of the moment of inertia (made by changing the car load) does not change the character of the movement (the movement remains stable in all the analyzed cases) but essentially alters the motion parameters both by the magnitude of the angular velocity and the lateral velocity, especially by the amplitude of the lateral wheel force.

A variation for inertia less than 10% leads to an increase for lateral forces to 280%, reaching values comparable to the grip force, resulting in vehicle skidding and loss of stability.

These conclusions refer to the simplified plan layout where the center of the table is placed in the plane of the tread and on the symmetry axis of the car. If calculation possibilities allow the analysis of some spatial models of the vehicle, including other physical and geometric parameters (unsuspended masses and suspended mass, rigidity of tires and suspension), then the moment of inertia taken into account for both suspended mass and non-suspended masses, can influence the character of the movement in a considerable manner.

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